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Bank-to-Turn Guidance Law Using Lyapunov Function and Nonzero Effort Miss

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A guidance law that directly computes the pitch acceleration and roll angle commands for Bank-To-Turn missiles is presented. The nonzero effort miss is introduced, and a Lyapunov function is defined in terms of nonzero effort miss. Lyapunov's stability theorem is used to obtain a guidance law that completely eliminates the trigonometric polar conversion, which conversion is necessary when the guidance commands and the input to the pitch and the roll autopilots are given in different coordinate systems. When the new guidance law is used, the missile tends to maintain its acceleration command above a certain level during its flight and thereby avoid a mathematical singularity that arises when a trigonometric inverse function is used to compute the roll command. A representative engagement scenario is used to demonstrate the effectiveness of the proposed guidance law. Numerical simulation results are compared with previous results and with results obtained using a proportional navigation guidance law that uses a polar conversion.

I. Introduction

A BANK-TO-TURN (BTT) missile is known to have advantages over the conventional Skid-To-Turn (STT) missile in terms of high lift, low drag, savings in storage, etc. These advantages are achieved by limiting the acceleration maneuver of a BTT missile to its maximum lift direction, which is to the pitch plane. BTT control is often called the preferred orientation control.[1] The potentials and limits of using BTT controls in terminal homing missiles are well documented in the pertinent literature.[1–3]

A BTT missile changes its flight direction by rolling and pitching motions, much like a conventional aircraft. Problems of design and analysis of BTT control have been an active subject of many investigations. The gyroscopic and aerodynamic cross-coupling arising when the BTT missile maneuvers simultaneously in pitch and roll directions produces unwanted sideslip and makes tracking the reference commands difficult.[4–7] However, the BTT guidance issue seems not have been addressed to an appropriate extent.

For the roll stabilized STT missile, the Cartesian guidance commands in terms of the pitch and yaw accelerations are directly fed into the autopilot command logic. In many circumstances, a STT guidance scheme is combined with a BTT autopilot, which autopilot controls the roll motion for coordination and small sideslip angle.[1] If a BTT control accepts polar commands, i.e., pitch acceleration and roll angle commands, and the guidance law computes its commands in different coordinates, for example, Cartesian coordinates, then the Cartesian commands must be converted into the polar commands. This procedure is called polar converting logic (PCL). The arctangent function is often used to compute the roll command.[2,3,8–10] However, using the arctangent function has the disadvantage that a mathematical singularity is introduced when the magnitude of the guidance command becomes zero. In Ref. 11, such situations were shown to happen when the missile is on the collision course, and this situation leads to system uncontrollability. Also, the roll motion becomes more susceptible to noise as the guidance command gets smaller.[2]

Published results related to the design of BTT guidance laws that directly compute the pitch acceleration and roll angle commands are relatively few.[10–13] In Refs. 11 and 12, a singular perturbation technique was used to obtain pitch acceleration and roll rate commands. Lin and Lee[13] have used the optimal control approach to obtain the guidance commands. However, the cited references still used the trigonometric inverse function somewhere in their solution procedures, and the authors assumed that the pitch and the roll autopilots have zero lag. Furthermore, a constant bias in pitch acceleration command had to be introduced to avoid singularity.

The main objective of this paper is to present a design method that completely eliminates the need for polar conversion. The guidance commands are expressed directly in the form of pitch acceleration and bank angle commands. For this purpose, the concept of nonzero effort miss (NZEM) is introduced, which concept is a natural extension of zero effort miss (ZEM). The NZEM provides a performance measure of missile maneuver not only at the intercept but also during the engagement. A Lyapunov function is defined in terms of NZEM, and then a guidance law based on the Lyapunov stability theorem is selected. Results of numerical simulation based on the simple missile–target kinematics are presented and compared with the previous ones and those obtained using a proportional navigation guidance law that uses a polar conversion.

II. Missile–Target Kinematics and Autopilot Models

Refer to Figs. 1 and 2. Equations (1–4) describe the relative kinematics between the missile and the target in the nonrotating reference frame[11–13]:

$$\dot{y}_r = v_y \tag{1}$$

$$\dot{z}_r = v_z \tag{2}$$

$$\dot{v}_y = a_y^T - a_m \sin\phi \tag{3}$$

$$\dot{v}_z = a_z^T + a_m \cos\phi \tag{4}$$

First-order lag models are adopted to represent the pitch and the roll autopilots:

$$\dot{a}_m = (1/\tau_a)(a_c - a_m) \tag{5}$$

$$\dot{\phi} = (1/\tau_\phi)(\phi_c - \phi) \tag{6}$$

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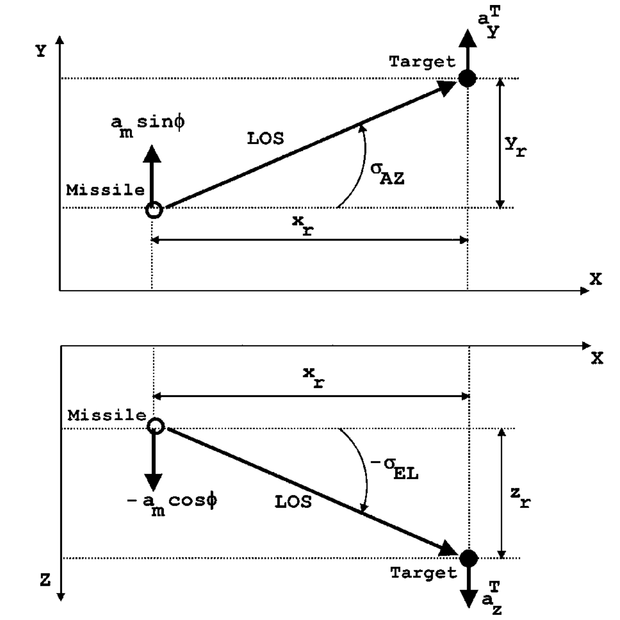

Fig. 1 Relative kinematics of missile-target engagement.

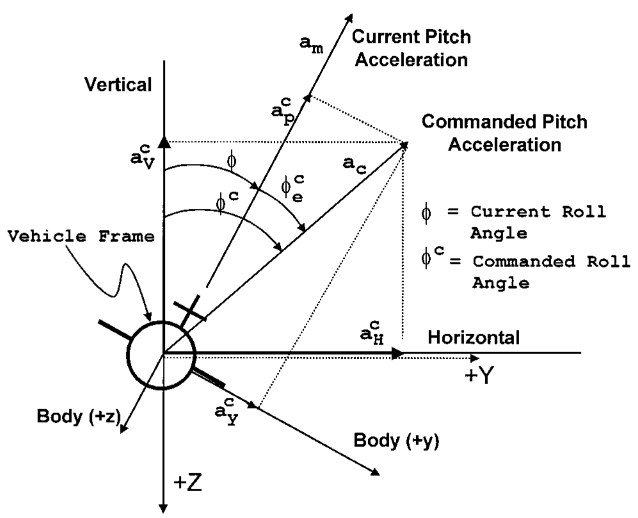

Fig. 2 Geometry of BTT guidance commands.

In Eqs. (1–6), (y_r, z_r) and (v_y, v_z), respectively, denote the relative position and velocity errors; (a_y^T, a_z^T) the components of lateral acceleration of the target in the nonrotating frame (the x-axis of the nonrotating reference frame is aligned with the initial line of sight direction); a_m the missile pitch acceleration; ϕ the missile roll angle, and τ_a and τ_ϕ the time constants of the pitch and the roll autopilots. With this model, it is assumed that the missile has a perfect coordinated turn capability for a BTT maneuver. The pitch acceleration command a_c and roll angle command ϕ_c may be obtained from a guidance law either after polar conversion if a STT guidance scheme is used, or directly from the BTT guidance scheme, as will be explained later.

III. STT Guidance Law with Polar Conversion

If a classical proportional navigation guidance (PNG) is used, then the guidance commands (a_H^c, a_V^c) in the nonrotating fixed frame may be written as

$$a_H^c = N_{\mathrm{PNG}}\dot{\sigma}_{\mathrm{AZ}} V_c \tag{7}$$

$$a_V^c = N_{\mathrm{PNG}}\dot{\sigma}_{\mathrm{EL}} V_c \tag{8}$$

where N_{PNG} is the navigation constant; V_c denotes the closing velocity; and $\dot{\sigma}_{\mathrm{AZ}}$ and $\dot{\sigma}_{\mathrm{EL}}$ are, respectively, the rates of the inertial azimuth and elevation angles of the line-of-sight as shown in Fig. 1.

For the terminal engagement, the classical or augmented PNG may be implemented using the concept of ZEM:

$$a_H^c = N_{\mathrm{PNG}}\left(1/t_{\mathrm{go}}^2\right) M_y^{\mathrm{ZEM}} \tag{9}$$

$$a_V^c = -N_{\mathrm{PNG}}\left(1/t_{\mathrm{go}}^2\right) M_z^{\mathrm{ZEM}} \tag{10}$$

where M_y^{ZEM} and M_z^{ZEM} are the components of ZEM along the y- and z-axes of the nonrotating frame, respectively.[14,15] Refer to Fig. 1. The equations for these parameters may be written as

$$M_y^{\mathrm{ZEM}} = y_r + v_y t_{\mathrm{go}} + \tfrac{1}{2} a_y^T t_{\mathrm{go}}^2 \tag{11}$$

$$M_z^{\mathrm{ZEM}} = z_r + v_z t_{\mathrm{go}} + \tfrac{1}{2} a_z^T t_{\mathrm{go}}^2 \tag{12}$$

where t_{go} is the time to go until intercept and is assumed to be

$$t_{\mathrm{go}} = t_f - t \tag{13}$$

and t_f denotes the flight time. For the terminal phase, the flight time may be approximated by $t_f = R/V_c$ where R is the initial range and V_c is the closing velocity.

A typical polar conversion is adopted from Refs. 3 and 8. Refer to Fig. 2. If we write the guidance commands in the rotating body frame

$$a_p^c = a_V^c \cos\phi + a_H^c \sin\phi \tag{14}$$

$$a_y^c = -a_V^c \sin\phi + a_H^c \cos\phi \tag{15}$$

then, the error in roll angle command can be determined as follows:

$$\phi_e^c = \tan^{-1}\left(a_y^c / a_p^c\right) \tag{16}$$

The inertial roll angle command for the roll channel is given by

$$\phi^c = \phi + \phi_e^c \tag{17}$$

The pitch acceleration command a_p^c is used as the reference input a_c to the pitch channel. The yaw acceleration of the missile will be driven to zero if the missile performs a coordinated turn.

Some missiles do not allow a negative angle of attack in order to avoid the possibility of the engine flame going out. Then the missile must roll up to ±180 deg for its pitch down maneuver. For ±180 deg roll capability, a four quadrant arctangent function must be used in implementing Eq. (16). The pitch acceleration command must remain positive:

$$a_c = \sqrt{a_y^{c^2} + a_p^{c^2}} \tag{18}$$

IV. BTT Guidance Law Design

A. Nonzero Effort Miss Distance

In this paper, we expand the concept of ZEM to include the relative acceleration between the missile and target and define the NZEM distance as follows:

$$M_y^{\mathrm{NZEM}} = y_r + v_y t_{\mathrm{go}} + \tfrac{1}{2} \dot{v}_y t_{\mathrm{go}}^2 \tag{19}$$

$$M_z^{\mathrm{NZEM}} = z_r + v_z t_{\mathrm{go}} + \tfrac{1}{2} \dot{v}_z t_{\mathrm{go}}^2 \tag{20}$$

Hereinafter, M_y and M_z are used to denote M_y^{NZEM} and M_z^{NZEM}, respectively.

Similar to the ZEM, the NZEM distance is the expected miss distance at intercept if both the missile and target maintain their respective accelerations. The concept of NZEM naturally indicates the performance of missile maneuver. Zero NZEM implies that the missile will hit the target if the current missile acceleration maneuver is maintained. Instead of staying on a straight collision course, the missile will keep moving on a curved collision course.

B. Derivation of the Guidance Law

Lyapunov's stability theorem provides an easy way of obtaining a control law for many dynamic systems. This approach was also used in the performance analysis of PNG type guidance laws.[16,17] For example, the Lyapunov function,

$$V = \tfrac{1}{2}\left[\left(M_y^{\mathrm{ZEM}^2} + M_z^{\mathrm{ZEM}^2}\right)\big/t_{\mathrm{go}}^2\right] \tag{21}$$

was used to investigate the finite time stability of a PNG guidance loop, and the instability of guidance loop was defined as the divergence of the Lyapunov function.[17]

In this study, we use the NZEM to define a Lyapunov function,

$$V = \tfrac{1}{2}\left(M_y^2 + M_z^2\right) \tag{22}$$

Differentiation of Eq. (22) yields

$$\frac{\mathrm{d}V}{\mathrm{d}t} = M_y\dot{M}_y + M_z\dot{M}_z = M_y\left(\dot{y}_r + \dot{v}_y t_{\mathrm{go}} - v_y + \frac{1}{2}\ddot{v}_y t_{\mathrm{go}}^2 - \dot{v}_y t_{\mathrm{go}}\right)$$
$$+ M_z\left(\dot{z}_r + \dot{v}_z t_{\mathrm{go}} - v_z + \frac{1}{2}\ddot{v}_z t_{\mathrm{go}}^2 - \dot{v}_z t_{\mathrm{go}}\right) \tag{23}$$

Substitution of Eqs. (1–4) into Eq. (23) gives

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{2}\ddot{v}_y t_{\mathrm{go}}^2 M_y + \frac{1}{2}\ddot{v}_z t_{\mathrm{go}}^2 M_z \tag{24}$$

Now, Eq. (24) may be further expanded using Eqs. (5–6) to get

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{2}t_{\mathrm{go}}^2[(M_y\sin\phi - M_z\cos\phi)\dot{a}_m$$
$$+ a_m\dot{\phi}(M_y\cos\phi + M_z\sin\phi)] \tag{25}$$

Equation 25 assumes that the target acceleration, if any, is constant. Note that from Fig. 2, we may define the NZEM in the rotating body frame as

$$M_a = M_y\sin\phi - M_z\cos\phi \tag{26}$$

$$M_\phi = M_y\cos\phi + M_z\sin\phi \tag{27}$$

where M_a is the component of NZEM along the body pitch direction ($-z$ body) and M_ϕ the component along the roll direction ($+y$ body). One may easily verify the relationship

$$V = \tfrac{1}{2}\left(M_y^2 + M_z^2\right) = \tfrac{1}{2}\left(M_a^2 + M_\phi^2\right) \tag{28}$$

Then, Eq. (25) is simply rewritten as

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{2}t_{\mathrm{go}}^2(M_a\dot{a}_m + M_\phi a_m\dot{\phi}) \tag{29}$$

When equations for the pitch and the roll autopilots, Eqs. (5) and (6), are substituted in Eq. (29), we get

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{2}t_{\mathrm{go}}^2\left[\frac{1}{\tau_a}(a_c - a_m)M_a + \frac{a_m}{\tau_\phi}(\phi_c - \phi)M_\phi\right] \tag{30}$$

In order to use Lyapunov's stability theorem, we need to assure the negative definiteness of Eq. (30).[18] We choose to use

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -NV = -\frac{1}{2}N\left(M_a^2 + M_\phi^2\right) \tag{31}$$

where N is the positive constant navigation gain that determines the rate of decrease of the Lyapunov function V. There may be other ways to make Eq. (31) always be negative, But this approach appears to be one of the simpler choices. Next, we select appropriate a_c and ϕ_c such that they satisfy Eqs. (30) and (31). Among several alternatives in choosing a_c and ϕ_c, we pick the following expression,

$$a_c = a_m + \left(N\big/t_{\mathrm{go}}^2\right)\tau_a M_a \tag{32}$$

$$\phi_c = \phi + \left(N\big/t_{\mathrm{go}}^2\right)(\tau_\phi/a_m)M_\phi \tag{33}$$

With zero lag autopilot models, after following the same procedure, one may easily find that the guidance commands are

$$a_c = a_c(t_0) + \int_{t_0}^{t}\frac{N}{t_{\mathrm{go}}^2}M_a\,\mathrm{d}t \tag{34}$$

$$\phi_c = \phi(t_0) + \int_{t_0}^{t}\frac{N}{t_{\mathrm{go}}^2}\frac{1}{a_c}M_\phi\,\mathrm{d}t \tag{35}$$

In Eqs. (34) and (35), $a_c(t_0)$ is an arbitrary bias in the pitch acceleration command.

An attractive feature of the proposed guidance law is that it does not require the use of a trigonometric inverse function to compute the roll angle command. From Eqs. (33) and (35), note that the roll command level is closely related to the pitch acceleration. Small pitch acceleration will induce relatively rapid roll motion, and vice versa. Since the new guidance law has a structure similar to PNG, it will be referred as a "BTT PNG" in this paper. Pitch acceleration and roll angle commands, respectively, are proportional to corresponding components of the NZEM distance. If any component of the NZEM is zero, then the missile will maintain its current acceleration level or bank angle.

Equation (31) may be used to predict the final performance in terms of the miss distance. First, rewrite Eq. (31):

$$\int_{V}^{V_f}\frac{1}{V}\,\mathrm{d}V = -N\int_{t}^{t_f}\mathrm{d}t \tag{36}$$

We obtain, after integration,

$$V(t_f) = Ve^{-N(t_f - t)} \tag{37}$$

Solving for the navigation gain N, we get

$$N = (1/t_{\mathrm{go}})\,\ln[V(t)/V_f] \tag{38}$$

Because the initial time is arbitrary, Eq. (38) determines a feedback type navigation gain for a given engagement situation in terms of the remaining flight time t_{go}, the current engagement condition $V(t)$, and the resultant desired intercept performance V_f. Note that V_f is directly related to the final miss at the intercept:

$$\sqrt{y_r(t_f)^2 + z_r(t_f)^2} = \sqrt{2V_f} \tag{39}$$

For a target performing zero or constant acceleration maneuver, Eq. (38) can be used either to determine the navigation gain for a desired final miss or to predict the final miss for a given navigation gain. However, the navigation gain will not be constant during the portion of time that the target performs different kinds of evasive maneuvers.

V. Simulation Examples

Two simulation examples are considered in this paper. The first example is taken from Ref. 11 in which the pitch acceleration and roll rate commands for BTT missiles are obtained using the optimal control approach. The initial relative positions and velocities are, respectively, 0 and 350 m/s in both the horizontal and vertical directions. The target performs a 9 g maneuver in both directions at the initiation of terminal engagement. In Ref. 11, a 10 g bias in pitch acceleration was used to ensure controllability of the missile. For the BTT PNG, Eqs. (34) and (35) are used because the zero lag models of the pitch and the roll autopilots are adopted. The pitch acceleration bias is varied from 10 to 0.1 g.

Figures 3–5 show the simulation results. Although the roll motions seem to be very similar, the acceleration command histories are quite different. Figures 3 and 4 support the expectation that the roll motion will be more rapid when the bias in pitch acceleration command is smaller. Note that the pitch acceleration command from the optimal BTT guidance approaches its bias command, but the BTT PNG maintains the acceleration level above the bias command. Interestingly, this trend is very similar to that of the classical PNG vs either the augmented PNG or other optimal guidance laws.[14,15,19]

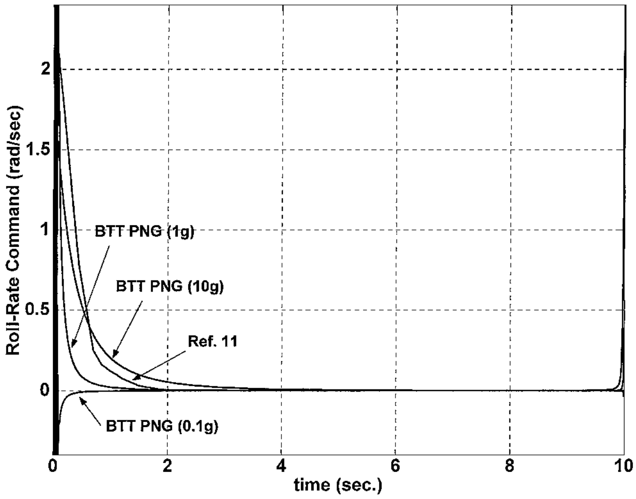

Fig. 3 Missile bank angle histories.

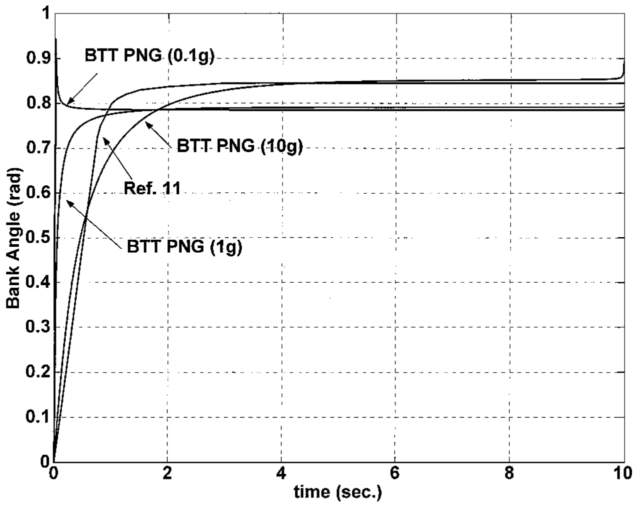

Fig. 4 Roll-rate command histories.

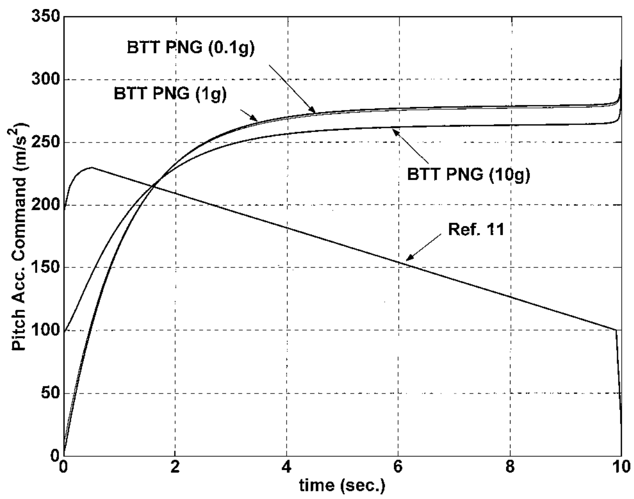

Fig. 5 Acceleration command histories.

The next example is the comparison of the BTT PNG to the classical proportional navigation guidance with polar conversion. Equations (7) and (8) are used to implement the classical PNG. For this implementation, the seeker is modeled as a first-order linear system, and its outputs are the rates of the inertial azimuth and elevation angles of the line-of-sight as shown in Fig. 1. These PNG commands are used in conjunction with Eqs. (14–18) for polar conversion. For a ± 180 deg roll capability of the missile, a four quadrant arctangent function is used in implementing Eq. (16).

Table 1 Simulation initial conditions and parameters

Description/Notation	Value	Unit
Initial condition		
x_r^0	8,200 (2,500)	ft (m)
y_r^0	0	
z_r^0	0	
v_x^0	−839 (256)	ft/s (m/s)
v_y^0	35 (11)	ft/s (m/s)
v_z^0	0	
Desired final miss distance	0.1 (0.03)	ft (m)
V_f	0.005 (5×10^{-4})	ft^2 (m^2)
Autopilot time constant		
τ_a	1.0	s
τ_ϕ	0.1	s
Seeker time constant		
τ_{AZ}	0.26	s
τ_{EL}	0.28	s
PNG navigation constant		
N_{PNG}	4	——

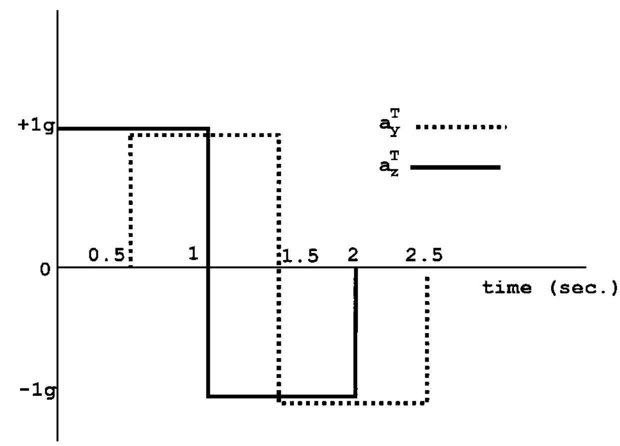

Fig. 6 Target acceleration profile.

As for the BTT PNG, the target acceleration is not included in calculating the NZEM because the classical PNG does not use this information. The relative velocities are reconstructed from the seeker outputs for the computation of NZEM.

Initial conditions and relevant parameters used in the simulation are summarized in Table 1. The simulation assumes that the missile has control over its longitudinal speed. The target performs a blind evasive maneuver with acceleration magnitude of a maximum of 1 g in both the y and z directions, and its acceleration profile is shown in Fig. 6.

Figures 7 and 8 show the time histories of the pitch acceleration and roll angle commands, respectively, during the flight. One can easily see that the missile that uses the BTT PNG law maintains almost a constant roll angle and a pitch acceleration above a certain level. However, the roll motion of the classical PNG with PCL becomes unstable as the commanded acceleration approaches zero. This instability is because the missile loses its controllability as it continues on the collision course.[11]

Once the missile is placed on the collision path, the guidance command from the classical PNG will become small, and the roll angle command obtained from the polar conversion will be more susceptible to noise. Therefore, maintaining the acceleration command above a certain level may help reduce such susceptibility. The trajectories of the missile and the target in the nonrotating frame are shown in Fig. 9. The trajectory resulting from the use of the BTT PNG law seems more curved than that of the classical PNG with PCL.

The time history of the Lyapunov function can be used to observe the guidance performance because the history contains the information related to the currently expected miss at the intercept, that is, the NZEM. As can be seen from Fig. 10, the Lyapunov function

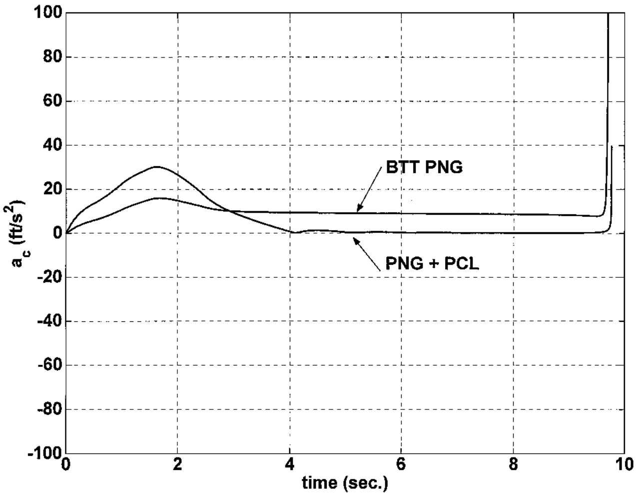

Fig. 7 Pitch acceleration command histories.

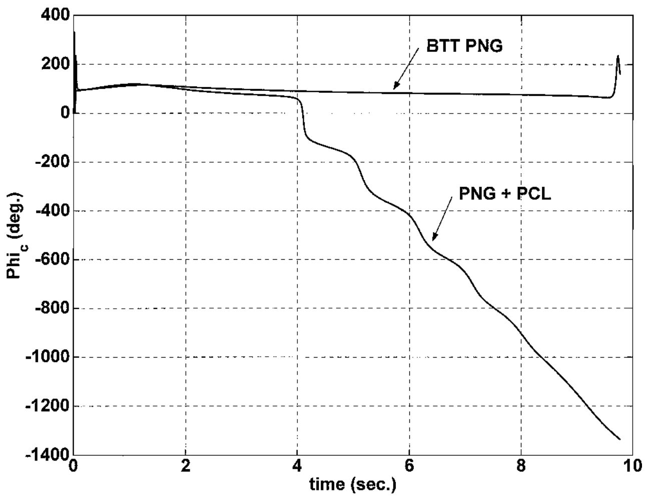

Fig. 8 Roll angle command histories.

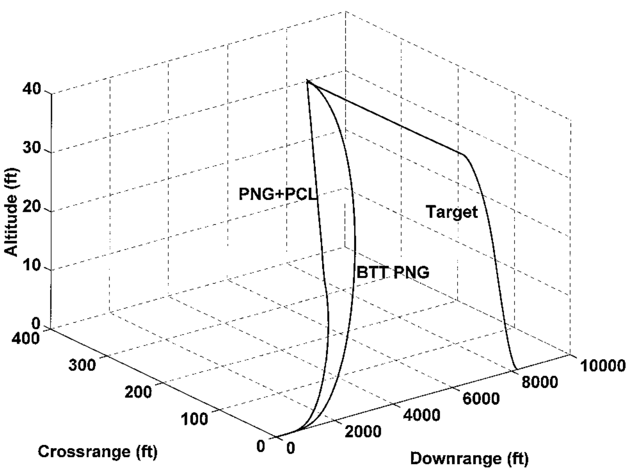

Fig. 9 Missile and target trajectories.

essentially decreases as the missile approaches the target. The initial increase in Lyapunov function is due to the seeker initialization and delay. Exponential decrease in the Lyapunov function of the new guidance law is the expected result because the law is designed so as to decrease the Lyapunov function. However, the guidance commands from the PNG and PCL seem not to produce a good performance during some periods of engagement because they increase the Lyapunov function during that time.

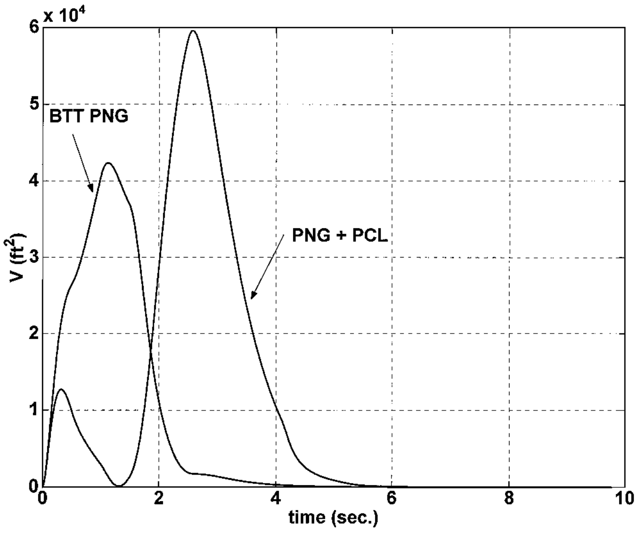

Fig. 10 Time history of the Lyapunov function.

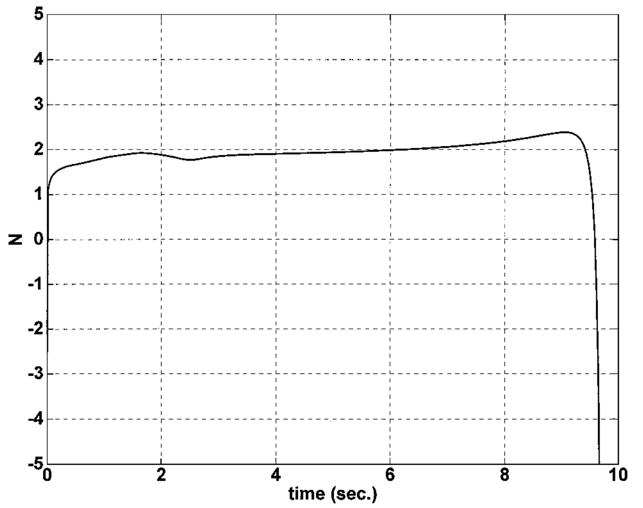

Fig. 11 Time history of the navigation gain.

Figure 11 shows the time history of the navigation gain computed using Eq. (38). Note that the gain remains almost constant for the portion of time that the target maneuver is constant. The reason for the gain not being an exact constant is that the BTT PNG is implemented using the outputs from the seeker modeled as a first-order lag system.

VI. Conclusions

In this paper, a new guidance law for BTT missiles is proposed. For this purpose, the concept of NZEM is introduced to form a Lyapunov function, and a guidance law based on Lyapunov stability theorem is derived. The resulting guidance commands are used as direct input to the pitch and the roll autopilots of BTT missiles. The new guidance scheme does not require the polar conversion that is typically needed when combining the STT guidance laws and the BTT controls. A useful relationship is obtained among the guidance gain, time to go, initial engagement scenario, and the final miss distance. When the new guidance law is used, the missile tends to maintain its pitch acceleration above a certain level and, therefore, stay on a curved flight path instead of a straight collision path. Staying on a current path has the benefit of reducing the susceptibility of roll motion to noise that may occur when the PNG type guidance law is used in conjunction with the polar conversion for the BTT missile guidance.

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